Optimization Techniques

Code: CS33123CS	Course Type: Elective	Semester: I
Evolution Scheme (TA-MSE-ESE): 20-30-100	Total Marks: 150	L-T-P (Credits): 3-0-0 (3)

Pre-Requisites: Programming Knowledge, Design and Analysis of Algorithm, Discrete Mathematics, Engineering Mathematics / Numerical Methods, Artificial Intelligence & Data Science

Course Objective: To equip students with fundamental and advanced **optimization techniques** for modeling and solving engineering problems, and to develop the ability to apply mathematical and computational methods for efficient decision-making.

Course Outcomes: At the end of the course, the student will be able to

CO-1	To develop a strong foundation in convex optimization and duality theory.
CO-2	To understand and analyse modern optimization algorithms.
CO-3	To apply optimization methods to machine learning, data science, and engineering problems.
CO-4	To explore both convex and non-convex optimization techniques.

Course Articulation Matrix:

PO	PEO-1	PEO-2	PEO-3	PEO-4	PEO-5
CO-1	3	2	1	-	-
CO-2	3	2	1	-	1
CO-3	3	2	-	-	3
CO-4	3	2	1	1	-

1 - Slightly; 2 - Moderately; 3 – Substantially

Syllabus:

Unit-1: Foundations of Optimization

Background Concepts: Convex sets and convex functions, Convex analysis fundamentals, Linear algebra and matrix theory (norms, eigenvalues, PSD matrices), Basics of probability theory for optimization

Preliminaries: Applications in engineering, AI, and data science, Formulation of optimization problems, Optimality conditions (first-order and second-order), Duality theory: Lagrangian, weak and strong duality, KKT conditions

Unit-2: First-Order Optimization Methods:

Gradient-Based Methods: Gradient descent and its variants, Convergence analysis Sub-gradient and

Proximal Methods: Sub-gradient methods for non-differentiable optimization, Proximal gradient methods, Regularization and composite optimization Advanced First-Order Techniques: Mirror descent, Multiplicative weights update method, Accelerated first-order methods (e.g., Nesterov acceleration)

Unit-3: Second-Order and Stochastic Optimization

Second-Order Methods: Newton's method, Quasi-Newton methods (BFGS, L-BFGS), Convergence and computational trade-offs

Stochastic Optimization: Stochastic optimization problem formulation, Notion of regret and performance measures, Online-to-batch conversion

Online Learning Methods: Online Gradient Descent (OGD), Exponentiated Gradient (EG), Online Mirror Descent (OMD), Methods with vanishing regret.

Unit-4: Non-Convex Optimization and Advanced Methods:

Non-Convex Optimization: Nature and challenges of non-convex problems, Multiple local minima and saddle points, Lack of global optimality guarantees. Applications: Sparse recovery, Affine rank minimization, Low-rank matrix completion. Solution Approaches: Convex relaxation methods. Non-convex methods (Projected gradient descent, alternating minimization). Interior Point Methods (for large-scale convex problems), Coordinate Descent Methods, Optimization for Deep Learning (SGD and mini-batch methods), Parallel and Distributed Optimization (basic concepts), Variance-Reduced Methods (SVRG, SAGA)

Learning Resources:

Text Books:

1. Dimitri P. Bertsekas, Nonlinear Programming, 3rd Edition, Athena Scientific, 2016
2. S. Boyd and L. Vandenberghe, Convex Optimization, The Cambridge University Press, 2003.
3. S. Bubeck, Convex Optimization: Algorithms and Complexity, Foundations and Trends in Machine Learning, 8(3-4): 231-357, 2015.
4. S. Sra, S. Nowozin, and S. Wright (eds). Optimization for Machine Learning, The MIT Press, 2011.

References:

1. T. Hastie, R. Tibshirani and M. J. Wainwright, Statistical Learning with Sparsity: the Lasso and Generalizations, Chapman and Hall/CRC Press, 2015. http://web.stanford.edu/~hastie/StatLearnSparsity/
2. E. Hazan. Introduction to Online Convex Optimization. Draft, 2015. http://ocobook.cs.princeton.edu/ 3. Y. Nesterov, Introductory lectures on convex optimization, Kluwer-Academic, 2003.